

# $V A \tilde{V}$ correlator within the instanton vacuum model

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## Abstract

The correlator of vector and nonsinglet axial-vector currents in the external electromagnetic field is calculated within the instanton liquid model of QCD vacuum. In general the correlator has two Lorentz structures: longitudinal  $w_L$  and transversal  $w_T$  with respect to axial-vector index. Within the instanton model the saturation of the anomalous  $w_L$  structure is demonstrated. It is known that in the chiral limit the transversal structure  $w_T$  is free from perturbative corrections. In this limit within the instanton model we calculate the transversal invariant function  $w_T$  at arbitrary momentum transfer  $q$  and show the absence of power corrections to this structure at large  $q^2$ . Instead there arise the exponential corrections to  $w_T$  at large  $q^2$  reflecting nonlocal properties of QCD vacuum. The slope of  $w_T$  at zero virtuality, the QCD vacuum magnetic susceptibility of the quark condensate and its momentum dependence are estimated.

## 1 Introduction

Since discovery of anomalous properties [1, 2] of the triangle diagram (Fig. 1) with incoming two vector and one axial-vector currents [3] many new interesting results have been gained. Recently the interest to triangle diagram has been renewed due to the problem of accurate calculation of higher order hadronic contributions to muon anomalous magnetic moment via the light-by-light scattering process<sup>1</sup>. At low energies the dynamics of light-by-light scattering is nonperturbative, so one needs rather realistic QCD inspired model to find a solution with the lowest model sensitivity.

The light-by-light scattering amplitude with one photon real and another photon has the momenta much smaller than the other two, can be analyzed using operator product expansion (OPE). In this special kinematics the amplitude is factorized into the amplitude depending on the largest photon momenta and the triangle amplitude involving the axial current  $A$  and two electromagnetic currents (one soft  $\tilde{V}$  and one virtual  $V$ ). The corresponding triangle amplitude, which can be viewed as a mixing between the axial and vector currents in the external electromagnetic field, were considered recently in [5, 6]. It can be expressed in terms of the two independent invariant functions, longitudinal  $w_L$  and

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<sup>1</sup>See, *e.g.*, [4, 5] and references therein.

transversal  $w_T$  with respect to axial current index. In perturbative theory for massless quarks (chiral limit) one has for space-like momenta  $q$  ( $q^2 \geq 0$ )

$$w_L(q^2) = 2w_T(q^2) = \frac{2}{q^2}. \quad (1)$$

The appearance of the longitudinal structure is the consequence of the axial Adler-Bell-Jackiw anomaly [1, 2]. Because there are no perturbative (Fig. 1b) [7] and nonperturbative (Fig. 1c) [8] corrections to the axial anomaly, the invariant function  $w_L$  remains intact when interaction with gluons is taken into account. Nonrenormalization of the longitudinal part follows from the 't Hooft consistency condition [8], i.e. the exact quark-hadron duality. In QCD this duality is realized as a correspondence between the infrared singularity of the quark triangle and the massless pion pole in terms of hadrons. It was shown in [6] (see also [9]) that in nonsinglet channel the transversal structure  $w_T$  is also free from perturbative corrections. OPE analysis indicates that at large  $q$  the leading nonperturbative power corrections to  $w_T$  can only appear starting with terms  $\sim 1/q^6$  containing the matrix elements of the operators of dimension six [10]. Thus, the transversal part of the triangle with a soft momentum in one of the vector currents has no perturbative corrections nevertheless it is modified nonperturbatively.

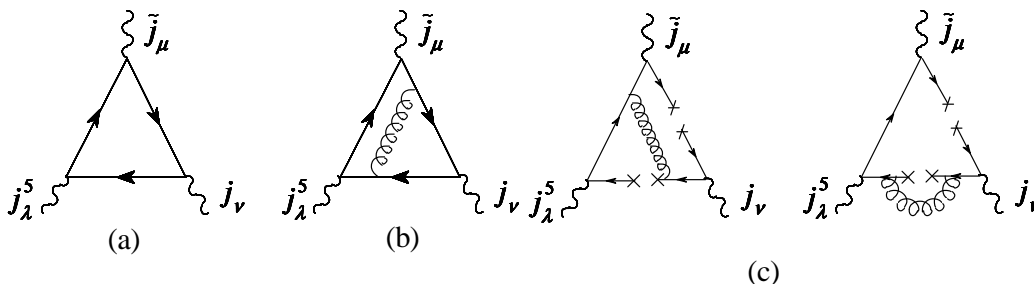


Figure 1: Quark triangle diagram, (a); perturbative gluon, (b), and four-quark condensate, (c), corrections to it.

In the present work we analyze in the framework of the instanton liquid model [11] the nonperturbative properties of the triangle diagram in the kinematics specified above (see Section 2 for further details). The model is based on the representation of QCD vacuum as an ensemble of strong vacuum fluctuations of gluon field, instantons. They characterize nonlocal properties of QCD vacuum [12, 13, 14]. The interaction of light  $u, d$  quarks in the instanton vacuum can be described in terms of effective 't Hooft four-quark action with nonlocal kernel induced by quark zero modes in the instanton field (Section 3). The gauged version of the model [15, 16, 17] meets the symmetry properties with respect to external gauge fields (Section 4), and corresponding vertices satisfy the Ward-Takahashi identities. Below in Section 5 we demonstrate how the anomalous structure  $w_L$  is saturated within the instanton liquid model. We also calculate the transversal invariant function  $w_T$  at arbitrary  $q$  and show that within the instanton model at large  $q^2$  there are no power corrections to this structure. The nonperturbative corrections to  $w_T$  at large  $q^2$  have exponentially decreasing behavior related to the short distance properties of the instanton nonlocality in the QCD vacuum. We also estimate the slope of transversal invariant function at zero virtuality.

When light quark current masses,  $m_f$  ( $f = u, d$ ), are switched on, additional OPE structures appear, with the leading one being of dimension four  $\sim m_f \bar{q} \sigma_{\alpha\beta} q$ . Its matrix element between vacuum and soft photon state is proportional to the quark condensate magnetic susceptibility introduced in Ref. [18]. Using the expansion of the triangle amplitude in inverse powers of momentum transfer squared we will derive an expression for the magnetic susceptibility in the instanton model and find its momentum dependence (Section 6).

## 2 The structure of $VA\tilde{V}$ correlator

We will employ a tensor decomposition of the  $VVA$  triangle graph amplitude suggested originally by Rosenberg [3] for the general kinematics of incoming momenta

$$T_{\mu\nu\lambda}(q_1, q_2) = A_1 q_1^\rho \varepsilon_{\rho\mu\nu\lambda} + A_2 q_2^\rho \varepsilon_{\rho\mu\nu\lambda} + A_3 q_1^\nu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\mu\lambda} + A_4 q_2^\nu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\mu\lambda} + A_5 q_1^\mu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\nu\lambda} + A_6 q_2^\mu q_1^\rho q_2^\sigma \varepsilon_{\rho\sigma\nu\lambda}, \quad (2)$$

where  $q_1$  and  $q_2$  are the vector field momenta with corresponding Lorentz indices  $\mu$  and  $\nu$ . The coefficients  $A_j = A_j(q_1, q_2)$ ,  $j = 1, \dots, 6$  are the Lorentz invariant amplitudes. The vector Ward identities provide a gauge invariant definition of the  $A_1$  and  $A_2$  amplitudes in terms of finite amplitudes  $A_k$ ,  $k = 3, \dots, 6$ ,

$$A_1 = (q_1 q_2) A_3 + q_2^2 A_4, \quad A_2 = (q_1 q_2) A_6 + q_1^2 A_5. \quad (3)$$

In the specific kinematics when one photon ( $q_2 \equiv q$ ) is virtual and another one ( $q_1$ ) represents the external electromagnetic field and can be regarded as a real photon with the vanishingly small momentum  $q_1$  only two invariant functions survive in linear in small  $q_1$  approximation [19]. It is convenient to define longitudinal and transversal with respect to axial current index amplitudes [6]

$$w_L(q^2) = 4\pi^2 \tilde{A}_4(q^2), \quad w_T(q^2) = 4\pi^2 \left( \tilde{A}_4(q^2) + \tilde{A}_6(q^2) \right), \quad (4)$$

where tilted amplitudes are  $\tilde{A}(q^2) \equiv A(q_1 = 0, q_2 = q)$ . In terms of  $w$  invariant functions the  $VA\tilde{V}$  amplitude becomes

$$\tilde{T}_{\mu\nu\lambda}(q_1, q_2) = \frac{1}{4\pi^2} \left[ w_T(q_2^2 q_1^\rho \varepsilon_{\rho\mu\nu\lambda} - q_2^\nu q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\lambda} + q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu}) - w_L q_2^\lambda q_1^\rho q_2^\sigma \varepsilon_{\rho\mu\sigma\nu} \right]. \quad (5)$$

Both structures are transversal with respect to vector current,  $q_2^\nu T_{\nu\lambda} = 0$ . As for the axial current, the first structure is transversal with respect to  $q_2^\lambda$  while the second is longitudinal and thus anomalous.

The amplitude for the triangle diagrams (Fig. 1) can be written as a correlator of the axial current  $j_\lambda^5$  and two vector currents  $j_\nu$  and  $\tilde{j}_\mu$

$$T_{\mu\nu\lambda} = - \int d^4x d^4y e^{iqx -iky} \langle 0 | T \{ j_\nu(x) \tilde{j}_\mu(y) j_\lambda^5(0) \} | 0 \rangle, \quad (6)$$

where for light  $u$  and  $d$  quarks one has

$$j_\mu = \bar{q} \gamma_\mu Q q, \quad j_\lambda^5 = \bar{q} \gamma_\lambda \gamma_5 \tau_3 q,$$

the quark field  $q_f^i$  has color ( $i$ ) and flavor ( $f$ ) indices, the charge matrix is  $Q = \frac{1}{2} (\frac{1}{3} + \tau_3)$  and the tilted current is for the soft momentum photon vertex.

In the local theory the one-loop result for the invariant functions  $w_T$  and  $w_L$  is<sup>2</sup>

$$w_L^{1\text{-loop}} = 2 w_T^{1\text{-loop}} = \frac{2N_c}{3} \int_0^1 \frac{d\alpha \alpha(1-\alpha)}{\alpha(1-\alpha)q^2 + m_f^2}, \quad (7)$$

where the factor  $N_c/3$  is due to color number and electric charge. The analytical result for the triangle diagram with finite quark masses has been obtained in [20] by dispersion integral method. In the chiral limit,  $m_f = 0$ , one gets the result (1) (with additional factor  $N_c/3$ ).

When nonperturbative contributions to the triangle amplitude (Fig. 1c) are taken into account it was shown in [10] by using the OPE methods that at large Euclidean  $q^2$  the difference between the longitudinal and transversal parts,  $w_{LT} = w_L - 2w_T$ , starts in the chiral limit from leading,  $\sim 1/q^6$ , power behavior. The power terms are expected to contribute only into the transversal function  $w_T$ . Below we demonstrate that within the instanton liquid model in the chiral limit all allowed by OPE power corrections to  $w_T$  cancel each other and only exponentially suppressed corrections remain.

### 3 The instanton effective quark model

To study nonperturbative effects in the triangle amplitude  $\tilde{T}_{\mu\nu\lambda}$  at low and high momenta one can use the framework of the effective field model of QCD. In the low momenta domain the effect of the nonperturbative structure of QCD vacuum become dominant. Since invention of the QCD sum rule method based on the use of the standard OPE it is common to parameterize the nonperturbative properties of the QCD vacuum by using infinite towers of the vacuum expectation values of the quark-gluon operators. From this point of view the nonlocal properties of the QCD vacuum result from the partial resummation of the infinite series of power corrections, related to vacuum averages of quark-gluon operators with growing dimension, and may be conventionally described in terms of the nonlocal vacuum condensates [12, 13]. This reconstruction leads effectively to nonlocal modifications of the propagators and effective vertices of the quark and gluon fields.

The adequate model describing this general picture is the instanton liquid model of QCD vacuum describing nonperturbative nonlocal interactions in terms of the effective action [11]. Spontaneous breaking the chiral symmetry and dynamical generation of a momentum-dependent quark mass are naturally explained within the instanton liquid model. The nonsinglet and singlet  $V$  and  $A$  current-current correlators, the vector Adler function have been calculated in [17, 21, 22] in the framework of the effective chiral model with instanton-like nonlocal quark-quark interaction [16]. In the same model the pion transition form factor normalized by axial anomaly has been considered in [23] for arbitrary photon virtualities.

We start with the nonlocal chirally invariant action which describes the interaction of

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<sup>2</sup>Here and below the small effects of isospin violation is neglected, considering  $m_f \equiv m_u = m_d$ .

soft quark fields [16]

$$S = \int d^4x \bar{q}_I(x) [i\gamma^\mu D_\mu - m_f] q_I(x) + \frac{1}{2} G_P \int d^4X \int \prod_{n=1}^4 d^4x_n \cdot \\ \cdot f(x_n) [\bar{Q}(X - x_1, X) \Gamma_P Q(X, X + x_3) \bar{Q}(X - x_2, X) \Gamma_P Q(X, X + x_4)], \quad (8)$$

where  $D_\mu = \partial_\mu - iV_\mu(x) - i\gamma_5 A_\mu(x)$  and the matrix product  $(1 \otimes 1 + i\gamma_5 \tau^a \otimes i\gamma_5 \tau^a)$  provides the spin-flavor structure of the interaction. In Eq. (8)  $\bar{q}_I = (\bar{u}, \bar{d})$  denotes the flavor doublet field of dynamically generated quarks,  $G_P$  is the four-quark coupling constant, and  $\tau^a$  are the Pauli isospin matrices. The separable nonlocal kernel of the interaction determined in terms of form factors  $f(x)$  is motivated by instanton model of QCD vacuum.

In order to make the nonlocal action gauge-invariant with respect to external gauge fields  $V_\mu^a(x)$  and  $A_\mu^a(x)$ , we define in (8) the delocalized quark field,  $Q(x)$ , by using the Schwinger gauge phase factor

$$Q(x, y) = P \exp \left\{ i \int_x^y dz_\mu [V_\mu^a(z) + \gamma_5 A_\mu^a(z)] T^a \right\} q_I(y), \quad \bar{Q}(x, y) = Q^\dagger(x, y) \gamma^0, \quad (9)$$

where  $P$  is the operator of ordering along the integration path, with  $y$  denoting the position of the quark and  $x$  being an arbitrary reference point. The conserved vector and axial-vector currents have been derived earlier in [16, 17, 22].

The dressed quark propagator,  $S(p)$ , is defined as

$$S^{-1}(p) = i\hat{p} - M(p^2), \quad (10)$$

with the momentum-dependent quark mass found as the solution of the gap equation

$$M(p^2) = m_f + 4G_P N_f N_c f^2(p^2) \int \frac{d^4k}{(2\pi)^4} f^2(k^2) \frac{M(k^2)}{k^2 + M^2(k^2)}. \quad (11)$$

The formal solution is expressed as [15]

$$M(p^2) = m_f + (M_q - m_f) f^2(p^2), \quad (12)$$

with constant  $M_q \equiv M(0)$  determined dynamically from Eq. (11) and the momentum dependent  $f(p)$  is the normalized four-dimensional Fourier transform of  $f(x)$  given in the coordinate representation.

The nonlocal function  $f(p)$  describes the momentum distribution of quarks in the nonperturbative vacuum. Given nonlocality  $f(p)$  the light quark condensate in the chiral limit,  $M(p) = M_q f^2(p)$ , is expressed as

$$\langle 0 | \bar{q} q | 0 \rangle = -N_c \int \frac{d^4p}{4\pi^4} \frac{M(p^2)}{p^2 + M^2(p^2)}. \quad (13)$$

Its  $n$ -moment is proportional to the vacuum expectation value of the quark condensate with the covariant derivative squared  $D^2$  to the  $n$ th power

$$\langle 0 | \bar{q} D^{2n} q | 0 \rangle = -N_c \int \frac{d^4p}{4\pi^4} p^{2n} \frac{M(p^2)}{p^2 + M^2(p^2)}. \quad (14)$$

The  $n$ th moment of the quark condensate appears as a coefficient of Taylor expansion of the nonlocal quark condensate defined as [12]

$$C(x) = \left\langle 0 \left| \bar{q}(0) P \exp \left[ i \int_0^x A_\mu(z) dz_\mu \right] q(x) \right| 0 \right\rangle \quad (15)$$

with gluon Schwinger phase factor inserted for gauge invariance and the integral is over the straight line path. Smoothness of  $C(x)$  near  $x^2 = 0$  leads to existence of the quark condensate moments in the *l.h.s.* of (14) for any  $n$ . In order to make the integral in the *r.h.s.* of (14) convergent the nonlocal function  $f(p)$  for large arguments must decrease faster than any inverse power of  $p^2$ , *e.g.*, like some exponential<sup>3</sup>

$$f(p) \sim \exp(-\text{const} \cdot p^\alpha), \quad \alpha > 0 \quad \text{as} \quad p^2 \rightarrow \infty. \quad (16)$$

Note, that the operators entering the matrix elements in (14) and (15) are constructed from the QCD quark and gluon fields. The *r.h.s.* of (14) is the value of the matrix elements of QCD defined operators calculated within the effective instanton model with dynamical quark fields. Within the instanton model in the zero mode approximation the function  $f(p)$  depends on the gauge. It is implied [13, 14] that the *r.h.s.* of (14) corresponds to calculations in the axial gauge for the quark effective field. It is selected among other gauges because in this gauge the covariant derivatives become ordinary ones:  $D \rightarrow \partial$ , and the exponential in (15) with straight line path is reduced to unit. In particular it means that one uses the quark zero modes in the instanton field given in the axial gauge when define the gauge dependent dynamical quark mass. The axial gauge at large momenta has exponentially decreasing behavior and all moments of the quark condensate exist. In principle, to calculate the gauge invariant matrix element corresponding to the of *l.h.s.* of (14) it is possible to use the expression for the dynamical mass given in any gauge, but in that case the factor  $p^{2n}$  will be modified for more complicated weight function providing invariance of the answer<sup>4</sup>.

Furthermore, the large distance asymptotics of the instanton solution is also modified by screening effects due to interaction of instanton field with surrounding physical vacuum [24, 25]. To take into account these effects and make numerics simpler we shall use for the nonlocal function the Gaussian form

$$f(p) = \exp(-p^2/\Lambda^2), \quad (17)$$

where the parameter  $\Lambda$  characterizes the nonlocality size and it is proportional to the inverse average size of instanton in the QCD vacuum.

The important property of the dynamical mass (11) is that at low virtualities passing through quark its mass is close to constituent mass, while at large virtualities it goes to the current mass value. As we will see in Sect. 5 this property is crucial in obtaining the anomaly at large momentum transfer. The instanton liquid model can be viewed as an approximation of large- $N_c$  QCD where the only new interaction terms, retained after integration of the high frequency modes of the quark and gluon fields down to a nonlocality

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<sup>3</sup>Very similar arguments lead the author of [30] to the conclusion that finiteness of all transverse momenta moments of the quark distributions guarantees the exponential fall-off of the cross sections.

<sup>4</sup>If one would naively use the dynamical quark mass corresponding to popular singular gauge then one finds the problem with convergence of the integrals in (14), because in this gauge there is only powerlike asymptotics of  $M(p) \sim p^{-6}$  at large  $p^2$ .

scale  $\Lambda$  at which spontaneous chiral symmetry breaking occurs, are those which can be cast in the form of four-fermion operators (8). The parameters of the model are then the nonlocality scale  $\Lambda$  and the four-fermion coupling constant  $G_P$ .

## 4 Conserved vector and axial-vector currents

The quark-antiquark scattering matrix (Fig. 2) in pseudoscalar channel is found from the Bethe-Salpeter equation as

$$\hat{T}_P(q^2) = \frac{G_P}{1 - G_P J_{PP}(q^2)}, \quad (18)$$

with the polarization operator being

$$J_{PP}(q^2) = \int \frac{d^4 k}{(2\pi)^4} f^2(k) f^2(k+q) \text{Tr} [S(k) \gamma_5 S(k+q) \gamma_5]. \quad (19)$$

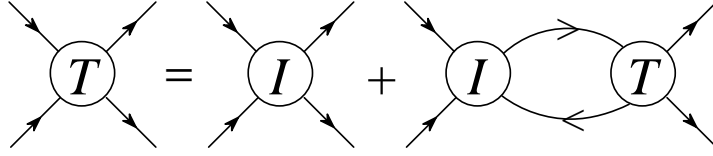


Figure 2: Diagrammatic representation of the Bethe-Salpeter equation for the quark-quark scattering matrix,  $T$ , with nonlocal instanton kernel,  $I$ .

The position of pion state is determined as the pole of the scattering matrix

$$\det(1 - G_P J_{PP}(q^2)) \Big|_{q^2 = -m_\pi^2} = 0. \quad (20)$$

The quark-pion vertex found from the residue of the scattering matrix is ( $k' = k + q$ )

$$\Gamma_\pi^a(k, k') = g_{\pi qq} i \gamma_5 f(k) f(k') \tau^a$$

with the quark-pion coupling found from

$$g_{\pi q}^{-2} = - \left. \frac{dJ_{ii}(q^2)}{dq^2} \right|_{q^2 = -m_\pi^2}, \quad (21)$$

where  $m_\pi$  is physical mass of the  $\pi$ -meson. The quark-pion coupling,  $g_{\pi q}^2$ , and the pion decay constant,  $f_\pi$ , are connected by the Goldberger-Treiman relation,  $g_\pi = M_q / f_\pi$ , which is verified to be valid in the nonlocal model [15], as requested by the chiral symmetry.

The vector vertex following from the model (8) is (Fig. 3a)

$$\Gamma_\mu(k, k') = \gamma_\mu + (k + k')_\mu M^{(1)}(k, k'), \quad (22)$$

where  $M^{(1)}(k, k')$  is the finite-difference derivative of the dynamical quark mass,  $q$  is the momentum corresponding to the current, and  $k$  ( $k'$ ) is the incoming (outgoing) momentum

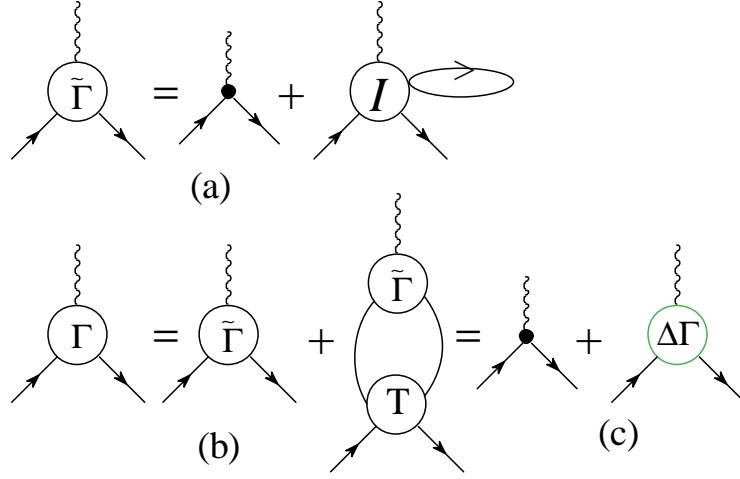


Figure 3: Diagrammatic representation of the bare (a) and full (b) quark-current vertices. Diagram (c) shows separation of local (fat dot) and nonlocal parts of the full vertex.

of the quark,  $k' = k + q$ . The finite-difference derivative of an arbitrary function  $F$  is defined as

$$F^{(1)}(k, k') = \frac{F(k') - F(k)}{k'^2 - k^2}. \quad (23)$$

The full axial vertex corresponding to the conserved axial-vector current is obtained after resummation of quark-loop chain that results in appearance of term proportional to the pion propagator [16] (Fig. 3b)

$$\begin{aligned} \Gamma_\mu^5(k, k') &= \gamma_\mu \gamma_5 + 2\gamma_5 \frac{q_\mu}{q^2} f(k) f(k') \left[ J_{AP}(0) - \frac{m_f G_P J_P(q^2)}{1 - G_P J_{PP}(q^2)} \right] \\ &+ (k + k')_\mu J_{AP}(0) \frac{(f(k') - f(k))^2}{k'^2 - k^2}, \end{aligned} \quad (24)$$

where we have introduced the notations

$$J_P(q^2) = \int \frac{d^4 k}{(2\pi)^4} f(k) f(k+q) \text{Tr} [S(k) \gamma_5 S(k+q) \gamma_5]. \quad (25)$$

$$J_{AP}(q^2) = 4N_c N_f \int \frac{d^4 l}{(2\pi)^4} \frac{M(l)}{D(l)} \sqrt{M(l+q) M(l)}. \quad (26)$$

The axial-vector vertex has a pole at

$$q^2 = -m_\pi^2 = m_c \langle \bar{q}q \rangle / f_\pi^2 \quad (27)$$

where the Goldberger-Treiman relation and definition of the quark condensate have been used. The pole is related to the denominator  $1 - G_P J_{PP}(q^2)$  in Eq. (24), while  $q^2$  in denominator is compensated by zero from square brackets in the limit  $q^2 \rightarrow 0$ . This compensation follows from expansion of  $J(q^2)$  functions near zero

$$\begin{aligned} J_{PP}(q^2) &= G_P^{-1} + m_c \langle \bar{q}q \rangle M^{-2}(0) - q^2 g_{\pi q}^{-2} + O(q^4), \\ J_{AP}(q^2 = 0) &= M(0), \quad J_P(q^2 = 0) = \langle \bar{q}q \rangle M^{-1}(0). \end{aligned}$$

In the chiral limit  $m_f = 0$  the second structure in square brackets in Eq. (24) disappears and the pole moves to zero.

The parameters of the model are fixed in a way typical for effective low-energy quark models. One usually fits the pion decay constant,  $f_\pi$ , to its experimental value, which in the chiral limit reduces to 86 MeV [26]. In the instanton model the constant,  $f_\pi$ , is determined by

$$f_\pi^2 = \frac{N_c}{4\pi^2} \int_0^\infty du \, u \frac{M^2(u) - uM(u)M'(u) + u^2M'(u)^2}{D^2(u)}, \quad (28)$$

where here and below  $u = k^2$ , primes mean derivatives with respect to  $u$ :  $M'(u) = dM(u)/du$ , etc., and

$$D(k^2) = k^2 + M(k)^2.$$

One gets the values of the model parameters [22]

$$M_q = 0.24 \text{ GeV}, \quad \Lambda_P = 1.11 \text{ GeV}, \quad G_P = 27.4 \text{ GeV}^{-2}. \quad (29)$$

## 5 $VA\tilde{V}$ correlator within the instanton liquid model

Our goal is to obtain the nondiagonal correlator of vector current and nonsinglet axial-vector current in the external electromagnetic field ( $VA\tilde{V}$ ) by using the effective instanton-like model (8). In this model the  $VA\tilde{V}$  correlator is defined by (Fig. 4a)

$$\begin{aligned} \tilde{T}_{\mu\nu\lambda}(q_1, q_2) = & -2N_c \int \frac{d^4k}{(2\pi)^4} \cdot \text{Tr} \left[ \Gamma_\mu(k + q_1, k) S(k + q_1) \Gamma_\lambda^5(k + q_1, k - q_2) S(k - q_2) \Gamma_\nu(k, k - q_2) S(k) \right], \end{aligned} \quad (30)$$

where the quark propagator, the vector and the axial-vector vertices are given by (10), (22) and (24), respectively. The structure of the vector vertices guarantees that the amplitude is transversal with respect to vector indices

$$\tilde{T}_{\mu\nu\lambda}(q_1, q_2)q_1^\mu = \tilde{T}_{\mu\nu\lambda}(q_1, q_2)q_2^\nu = 0 \quad (31)$$

and the Lorentz structure of the amplitude is given by (5).

It is convenient to express Eq. (30) as a sum of the contribution where all vertices are local (Fig. 4b), and the rest contribution containing nonlocal parts of the vertices (Fig. 3c). Further results in this section will concern the chiral limit.

The contributions of diagram 4b to the invariant functions at space-like momentum transfer,  $q^2 \equiv q_2^2$ , are given by

$$\tilde{A}_4^{(L)}(q^2) = \frac{N_c}{9q^2} \int \frac{d^4k}{\pi^4} \frac{1}{D_+^2 D_-} \left[ k^2 - 4 \frac{(kq)^2}{q^2} + 3(kq) \right], \quad (32)$$

$$\tilde{A}_6^{(L)}(q^2) = -\frac{1}{2} \tilde{A}_4^{(L)}(q^2). \quad (33)$$

where the notations used here and below are

$$k_+ = k, \quad k_- = k - q, \quad k_\perp^2 = k_+ k_- - \frac{(k_+ q)(k_- q)}{q^2},$$

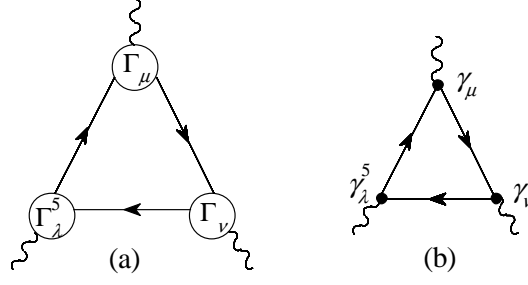


Figure 4: Diagrammatic representation of the triangle diagram in the instanton model with dressed quark lines and full quark-current vertices (a); and part of the diagram when all vertices are local one (b).

$$D_{\pm} = D(k_{\pm}^2), \quad M_{\pm} = M(k_{\pm}^2), \quad f_{\pm} = f(k_{\pm}^2).$$

At large  $q^2$  one has an expansion

$$\tilde{A}_4^{(L)}(q^2 \rightarrow \infty) = \frac{N_c}{6\pi^2} \left( \frac{1}{q^2} + \frac{a_{(4)}^{(L)}}{q^4} + \frac{a_{(6)}^{(L)}}{q^6} + O(q^{-8}) \right), \quad (34)$$

with coefficients given by

$$a_{(4)}^{(L)} = - \int_0^\infty du \frac{M^2(u)}{D^2(u)} (2u + M^2(u)), \quad a_{(6)}^{(L)} = - \frac{2}{3} \int_0^\infty du \frac{u M^2(u) (u + 2M^2(u))}{D^2(u)}. \quad (35)$$

It is clear that the contribution (32) saturate the anomaly at large  $q^2$ . The reason is that the leading asymptotics of (32) is given by the configuration where the large momentum is passing through all quark lines. Then the dynamical quark mass  $M(k)$  reduces to zero and the asymptotic limit of triangle diagram with dynamical quarks and local vertices coincides with the standard triangle amplitude with massless quarks and, thus, it is independent of the model.

The contribution to the form factors when the nonlocal parts of the vector and axial-vector vertices are taken into account is given by

$$\begin{aligned} \tilde{A}_4^{(NL)}(q^2) = \frac{N_c}{3q^2} \int \frac{d^4k}{\pi^4} \frac{1}{D_+^2 D_-} \left\{ M_+ \left[ M_+ - \frac{4}{3} M'_+ k_\perp^2 \right] - \right. \\ \left. - M^{2(1)}(k_+, k_-) \left( 2 \frac{(kq)^2}{q^2} - (kq) \right) \right\}. \end{aligned} \quad (36)$$

One has for the leading terms of large  $q^2$  asymptotics

$$\tilde{A}_4^{(NL)}(q^2 \rightarrow \infty) = \frac{N_c}{6\pi^2} \left( \frac{a_{(4)}^{(NL)}}{q^4} + \frac{a_{(6)}^{(NL)}}{q^6} + O(q^{-8}) \right), \quad (37)$$

with coefficients given by

$$\begin{aligned} a_{(4)}^{(NL)} &= 2 \int_0^\infty du \frac{u M(u)}{D^2(u)} (M(u) - u M'(u)), \\ a_{(6)}^{(NL)} &= \frac{2}{3} \int_0^\infty du \frac{u^3 M(u) M'(u)}{D^2(u)}. \end{aligned} \quad (38)$$

In sum of two contributions both power corrections with coefficients  $a_{(4)}$  and  $a_{(6)}$  are canceled. To prove cancellation for the  $a_{(4)}$  coefficient one needs to use integration by parts.

Summing analytically the local (32) and nonlocal (36) parts provides us with the result required by the axial anomaly

$$w_L(q^2) = 4\pi^2 \tilde{A}_4(q^2) = \frac{2N_c}{3} \frac{1}{q^2}. \quad (39)$$

Fig. 5 illustrates how different contributions saturate the anomaly. Note, that at zero virtuality the saturation of anomaly follows from anomalous diagram of pion decay in two photons. This part is due to the triangle diagram involving nonlocal part of the axial vertex and local parts of the photon vertices. The result (39) is in agreement with the statement about absence of nonperturbative corrections to longitudinal invariant function following from the 't Hooft duality arguments. Earlier this consistency has also been demonstrated within the QCD sum rules [27, 28] and within dispersion method [29] considering the lowest orders of expansion of the triangle diagram in condensates.

For  $\tilde{A}_6^{(NL)}(q^2)$  invariant function one gets

$$\begin{aligned} \tilde{A}_6^{(NL)}(q^2) = & -\frac{N_c}{6q^2} \int \frac{d^4k}{\pi^4} \frac{1}{D_+^2 D_-} \left\{ (M_+ + M_-) \left[ M_+ - \frac{(kq)}{q^2} (M_+ - M_-) \right] - \right. \\ & -\frac{2}{3} M'_+ \left[ 2k_\perp^2 M_+ - M_- \frac{q^2}{k_+^2 - k_-^2} \left( k^2 + 2 \frac{(kq)^2}{q^2} - 6(kq) \frac{k^2}{q^2} \right) \right] \Big\} + \\ & + \frac{2N_c}{9q^2} \int \frac{d^4k}{\pi^4} \frac{\sqrt{M_+ M_-}}{D_+^2 D_-} \frac{k_\perp^2}{k_+^2 - k_-^2} [M_+ - M_- - 2M'_+(kq)]. \end{aligned} \quad (40)$$

Then, let us consider the combination of invariant functions which show up nonperturbative dynamics

$$w_{LT}(q^2) \equiv w_L(q^2) - 2w_T(q^2) = -4\pi^2 \left[ \tilde{A}_4(q^2) + 2\tilde{A}_6(q^2) \right]. \quad (41)$$

From (33) we see that the contribution to  $w_{LT}$  from the triangle diagram 4b with local vertices is absent. In sum of  $\tilde{A}_4(q^2)$  and  $\tilde{A}_6(q^2)$  a number of cancellations takes place and the final result is quite simple

$$\begin{aligned} w_{LT}(q^2) = & \frac{4N_c}{3q^2} \int \frac{d^4k}{\pi^2} \frac{\sqrt{M_-}}{D_+^2 D_-} \left\{ \sqrt{M_-} \left[ M_+ - \frac{2}{3} M'_+ \left( k^2 + 2 \frac{(kq)^2}{q^2} \right) \right] - \right. \\ & \left. - \frac{4}{3} k_\perp^2 \left[ \sqrt{M_+} M^{(1)}(k_+, k_-) - 2(kq) M'_+ \sqrt{M}^{(1)}(k_+, k_-) \right] \right\}. \end{aligned} \quad (42)$$

The behavior of  $w_{LT}(q^2)$  is presented in Fig. 6.

In the above expression the integrand is proportional to the product of nonlocal form factors  $f(k_+) f(k_-)$  depending on quark momenta passing through different quark lines. Then, it becomes evident that the large  $q^2$  asymptotics of the integral is governed by the asymptotics of the nonlocal form factor  $f(q)$  which is exponentially suppressed (16). Thus, within the instanton model the distinction between longitudinal and transversal

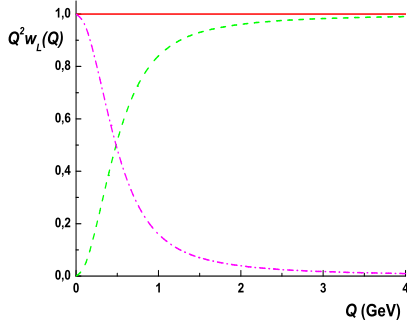


Figure 5: Normalized  $w_L$  invariant function constrained by ABJ anomaly from triangle diagram Fig. 4a (solid line) and different contributions to it: from local part, Fig. 4b, (dashed line), and from the nonlocal part (dash-dotted line).

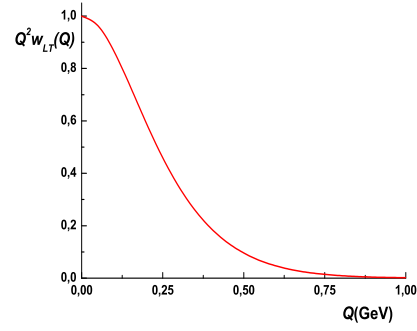


Figure 6: Normalized  $w_{LT}$  invariant function versus  $Q$  predicted by the instanton model from the diagram Fig. 4a.

parts is exponentially suppressed at large  $q^2$  and all allowed by OPE power corrections are canceled each other. Recently, it was proven that the relation

$$w_{LT}[m_f = 0] = 0, \quad (43)$$

which holds at the one-loop level gets no perturbative corrections from gluon exchanges [6]. The instanton liquid model indicates that it may be possible that due to anomaly this relation is violated at large  $q^2$  only exponentially.

We also find numerical values of the slope of invariant function  $w_{LT}$  at zero virtuality

$$\left. \frac{\partial w_{LT}(q^2)}{\partial q^2} \right|_{q^2=0} (\mu_{\text{Inst}}) = -3.8 \text{ GeV}^{-2}, \quad (44)$$

and its width

$$\lambda_{LT}^2 \equiv \int u w_{LT}(u) du \cdot \left( \int w_{LT}(u) du \right)^{-1} = 0.54 \text{ GeV}^2. \quad (45)$$

## 6 Magnetic susceptibility of quark condensate

In this section we consider the leading power corrections to  $w_L$  and  $w_T$  resulting from inclusion of current quark mass,  $m_f$ , into consideration. An appearance of this kind of power corrections is already clear from perturbative expression (7). In OPE the leading, by dimension, correction to the invariant functions  $w_{T,L}(q^2)$  is

$$\Delta w_L = 2 \Delta w_T = \frac{4m_f \kappa_f}{3q^4}, \quad (46)$$

where  $\kappa_f$  are the matrix element of dimension 3 operators

$$\mathcal{O}_f^{\alpha\beta} = -i \bar{q}_f \sigma^{\alpha\beta} \gamma_5 q^f, \quad (47)$$

between the soft photon and vacuum states. Proportionality to  $m_f$  in (46) is in correspondence with chirality arguments.

In perturbation theory the matrix element  $\kappa_f$  of the chirality-flip operator  $O_f$  is proportional to  $m_f$ . Nonperturbatively, however, due to spontaneous breaking of the chiral symmetry  $\kappa_f$  does not vanish at  $m_f = 0$ . It is convenient to introduce the magnetic susceptibility  $\chi_m$  normalized by the quark condensate [18]

$$\kappa_f = -4\pi^2 \langle \bar{q}q \rangle \chi_m.$$

This representation emphasizes that magnetic susceptibility for the nondiagonal vector-axial-vector correlator in the external electromagnetic field plays the similar role as the quark condensate for the diagonal correlators of vector and axial-vector currents.

In the instanton model the  $VA\tilde{V}$  correlator is given by (30) with the quark propagator, the vector and the axial-vector vertices defined by (10), (22) and (24), with the quark mass,  $m_f$ , being included. Keeping in the calculation only linear in the current quark mass terms one finds at large  $q^2$  the correction at twist 4 level (46) for the contribution of diagram Fig. 4b

$$\Delta\tilde{A}_4^{(L)}(q^2 \rightarrow \infty) = -\frac{1}{q^4} \frac{2m_f}{3} \frac{N_c}{\pi^2} \int du \frac{u^2 M(u)}{D^3(u)}, \quad (48)$$

and from nonlocal part

$$\Delta\tilde{A}_4^{(NL)}(q^2 \rightarrow \infty) = -\frac{1}{q^4} \frac{m_f}{3} \frac{N_c}{\pi^2} \int du \frac{uM(u)}{D^3(u)} [-u + 3M^2(u) - 4uM(u)M'(u)]. \quad (49)$$

The leading asymptotics linear in current mass for the invariant function  $\tilde{A}_6$  is given by the relation

$$\Delta\tilde{A}_6(q^2 \rightarrow \infty) = -\frac{1}{2}\Delta\tilde{A}_4(q^2 \rightarrow \infty) \quad (50)$$

which is in accordance with OPE (46).

Then, summing up contributions (48) and (49) and comparing the result at large  $q^2$  with OPE one gets the magnetic susceptibility in the form

$$\chi_m(\mu_{\text{Inst}}) = -\frac{1}{\langle 0|\bar{q}q|0\rangle} \frac{N_c}{4\pi^2} \int du \frac{u(M(u) - uM'(u))}{D^2(u)}, \quad (51)$$

where the quark condensate is defined in (13).

Alternatively, to get (51) we may simply calculate the matrix element of  $\mathcal{O}_f^{\alpha\beta}$  (47) between vacuum and one real photon state and use Eq. (22) for the quark-photon vertex. In this way it is easy to show that the result (51) stays unchanged when one includes the vector meson degrees of freedom. Indeed, in the extended model [22, 31] the vector vertex gets contribution from vector  $\rho$  and  $\omega$  mesons in the form

$$\Delta\Gamma_\mu^a(p, p') = \left( g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \gamma_\nu T^a \frac{G_V f^V(p) f^V(p')}{1 - G_V J_V^T(q^2)} B_V(q^2), \quad (52)$$

where  $T^a$  is a flavor matrix,  $f^V(p)$ ,  $J_V^T(q^2)$ ,  $G_V$  are the nonlocal form factor, the polarization operator and four-quark coupling in the vector channel, correspondingly. Due to

conservation of the vector current one has  $B_V(q^2 = 0) = 0$  and thus there is no contribution to the magnetic susceptibility.

It is easy also to derive the momentum dependence of the magnetic susceptibility

$$\chi_m(q) = -\frac{N_c}{\langle 0|\bar{q}q|0\rangle} \int \frac{d^4k}{4\pi^4} \frac{1}{D_+ D_-} \left\{ \left[ M_+ - \frac{kq}{q^2} (M_+ - M_-) \right] (1 + B_V(q^2) f_+^V f_-^V) - \right. \\ \left. - \frac{2}{3} k_\perp^2 M^{(1)}(k_+, k_-) \right\}, \quad (53)$$

presented in Fig. 7. At large  $q$  the integral in (53) is proportional to the quark condensate providing the asymptotic result

$$\chi_m(q \rightarrow \infty) = \frac{2}{q^2}. \quad (54)$$

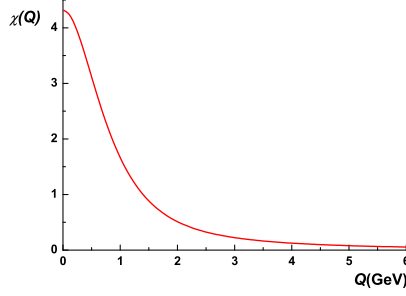


Figure 7: The momentum dependence of the magnetic susceptibility of quark condensate.

Recently, Eq. (51) has been obtained by more complicated way in [32]. Also note that the instanton model does not support use of the pion dominance for estimate of the magnetic susceptibility as it was attempted in [6]. The reason is that the pion pole in the axial vertex (24) is accompanied by the exponentially suppressed residue  $J_P(q^2)$ . Thus, it does not contribute to the twist 4 coefficient.

Given model parameters (29) one finds numerical values for the quark condensate and the magnetic susceptibility

$$\langle 0|\bar{q}q|0\rangle(\mu_{\text{Inst}}) = -(214 \text{ GeV})^3, \quad \chi_m(\mu_{\text{Inst}}) = 4.32 \text{ GeV}^{-2}, \quad (55)$$

where  $\mu_{\text{Inst}}$  is the normalization scale typical for instanton fluctuations. To leading-logarithmic accuracy scale dependence of these values is predicted by QCD as

$$\langle 0|\bar{q}q|0\rangle(\mu) = L^{-\gamma_{\bar{q}q}/b} \langle 0|\bar{q}q|0\rangle(\mu_0), \quad \chi_m(\mu) = L^{-(\gamma_0 - \gamma_{\bar{q}q})/b} \chi_m(\mu_0), \quad (56)$$

where  $L = \alpha_s(\mu)/\alpha_s(\mu_0)$ ,  $b = (11N_c - 2n_f)/3$ ,  $\gamma_{\bar{q}q} = -3C_F$  is the anomalous dimension of the quark condensate,  $\gamma_0 = C_F$  is the anomalous dimension of the chiral-odd local operator of leading-twist,  $C_F = 4/3$ . We may fix the normalization scale of the model by comparing the value of the condensate with that found in QCD sum rule at some standard

normalization point:  $\langle 0 | \bar{q}q | 0 \rangle (\mu_0 = 1 \text{ GeV}) = -(240 \text{ GeV})^3$ . Then one finds  $L = 2.17$  that corresponds to the normalization point  $\mu_{\text{Inst}} \approx 0.5 \text{ GeV}$ , with the QCD constant for three flavors being  $\Lambda_{\text{QCD}}^{(n_f=3)} = 296 \text{ MeV}$ . The rescaled magnetic susceptibility calculated in the model will be

$$\chi_m (\mu_{\text{Inst}} = 1 \text{ GeV}) = 2.73 \text{ GeV}^{-2}, \quad (57)$$

which is in rather good agreement with the latest numerical value of  $\chi_m$  obtained with the QCD sum rule fit [33]:  $\chi_m (\mu_{\text{SR}} = 1 \text{ GeV}) = (3.15 \pm 0.3) \text{ GeV}^{-2}$ . A phenomenology of hard exclusive processes sensitive to the magnetic susceptibility  $\chi_m$ , see [33], will possibly help to fix its value.

## 7 Conclusions

In the framework of the instanton liquid model we calculated for arbitrary momenta transfer the nondiagonal correlator of the vector and nonsinglet axial-vector currents in the background of a soft vector field. In this case we find that at large momenta the nonperturbative power corrections are absent in the chiral limit for the transversal part  $w_T$  of triangle diagram. The transversal part is corrected only by exponentially small terms which reflects the nonlocal structure of QCD vacuum. Within the instanton model the saturation of the anomalous, longitudinal  $w_L$  structure is demonstrated explicitly. Using the instanton liquid model we also derive an expression for the quark condensate magnetic susceptibility and its momentum dependence.

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